Quiz 3 solution
MTH 102 – Dr. Shaffer
November 10, 2006

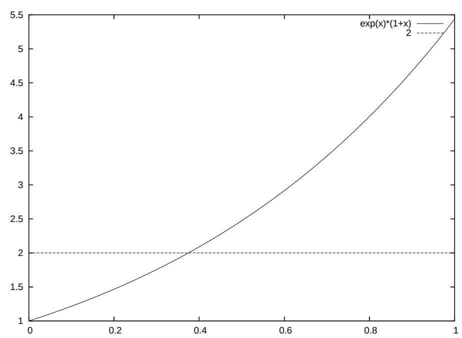
Instructions: This quiz is closed book/notes. The work on this quiz must be entirely your own as indicated in the course syllabus. Solve all problems and **show all of your work.**

1. (5 pts) Find all of the extrema (local and global) of the function $f(x) = 2x - xe^x$ defined over the interval $-3 \leq x \leq 3$. Give both the values of x and $f(x)$ at these extrema and label the extrema that are global.

$$
\begin{aligned}
f'(x) &= 2 - e^x - xe^x \\
f'(x) &= 2 - e^x(1 + x)
\end{aligned}
$$

$$
\begin{aligned}
f'(x_c) &= 0 \\
2 - e^{x_c}(1 + x_c) &= 0 \\
e^{x_c}(1 + x_c) &= 2
\end{aligned}
$$

Solve graphically by graphing $e^{x_c}(1 + x_c)$ and 2 and looking for where they cross:



We get approximately $x = 0.375$ (use the trace feature on your calculator). Now compute the second derivative to see if this is a max or a min.

$$
\begin{aligned}
f''(x) &= -2e^x - xe^x \\
f''(x_c) &= -3.4
\end{aligned}
$$

Since the second derivative is negative this critical point must be a local max. Now we compute the function value at this max and the two endpoints:

$$
\begin{aligned}
f(-3) &= -5.85 \\
f(3) &= -54.26 \\
f(0.375) &= 0.20
\end{aligned}
$$

So $x = 0.375$ is the position of the gobal max and $x = 3$ is the position of the global min.

2. Research shows that demand for i-pods is goverened by $q(p) = 300 - 0.15p$ where p is the price and $q(p)$ is the number sold per year at price p. IPods incur a fixed production cost of $10000 and a variable cost of 32 per unit.

   (a) (1 pts) Give the cost function $C(q)$.

   $C(q) = 10000 + 32q$

   (b) (1 pts) Give the revenue function $R(q)$ for selling q units at a price p.

   $R(q) = pq$

   (c) (3 pts) Using the deman equation, rewrite $C(q)$ and $R(q)$ as functions of p.

$$\begin{aligned} C(p) &= 10000 + 32(300 - 0.15p) \\ C(p) &= 19600 - 4.8p \\ \\ R(p) &= p(300 - 0.15p) \\ R(p) &= 300p - 0.15p^2 \end{aligned}$$

   (d) (5 pts) Give the profit as a function of price $\pi(p)$.

$$\begin{aligned} \pi(p) &= R(p) - C(p) \\ \pi(p) &= 300 - p0.15p^2 - (19600 - 4.8p) \\ \pi(p) &= -19600 + 304.8p - 0.15p^2 \end{aligned}$$

   (e) (5 pts) What price maximizes profit?

$$\begin{aligned} \pi'(p) &= 304.8 - 0.30p \\ \pi'(p_c) &= 0 \\ 304.8 - 0.30p_c &= 0 \\ p_c &= 1016 \end{aligned}$$

   Verify that this is a maximum by either taking the second derivative and checking its sign or by noticing that $\pi(p)$ is quadratic (graph is a parabola) with a negative leading coefficient so it must have only a maximum.